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Analysis of the Structural Vulnerability of the Interconnected Power Grid of Continental Europe with IPS/UPS Based on Extended Topological Approach

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Abstract

Power systems as one of the key infrastructures play a crucial role in any country's economy and social life. A large-scale blackout can affect all sectors in a society such as industrial, commercial, residential and essential public services. However, the frequency of large-scale blackouts across the world is not being reduced though advanced technology and huge investment have been applied into power systems. Given a single blackout it is possible to analyze the causes with the traditional engineering methods. What we want to do instead is not to explain the causes of blackouts but to find what are the most critical elements of the power system, in order to improve the resilience of the system itself. As blackout can happen in different load conditions, we do not want a method which depends on the load/generation level. We want a method independent from these factors: this is the structural perspective.

When the interconnection between European and Russian power grids will create the largest interconnected power grid throughout the world in terms of the scale, transmission distance and involved countries, analyzing the vulnerability of the large-scale power grid will be useful to maintain its reliable and secure operation. To analyze the vulnerability of the interconnected power grid, in this paper we firstly create the interconnected transmission network between continental Europe and Commonwealth of Independent States (CIS) and Baltic countries; then, the structural vulnerability of the interconnected power grid is analyzed from a topological point of view using our proposed extended topological method which incorporates some electrical engineering characteristics into complex network methodology. We find that these power grids of continental Europe, Baltic states and CIS countries can benefit from the interconnection since the interconnected power grid cannot only improve the overall network performance of these power grids in Baltic states and CIS countries but also increase their structural robustness.

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Key words: power grid; structural vulnerability; extended topological method; structural robustness;

1. Introduction

The Power grid is a key infrastructure for modern society. However, the frequency of large-scale blackouts across the world is not being reduced, especially in the United States, even though advanced technology and huge investments have been utilized in maintaining reliability and security of the system [1]. The series of blackouts seems to show that existing analysis techniques in electrical engineering are not easy tools to understand power systems because of their complexity [2]. This complexity is not just due to interwoven and intricate topology consisting of a multitude of buses and lines but also from complicated decision-making of system operators that maintain instant power balance of generators and loads in large-scale transmission networks across a multitude of countries in order to guarantee the security and reliability of the system. It is noticed that there is a strong link between topological structure and operation performance in power systems because the structural change could alter operational conditions of a power system and then change its operation performance. As a result, there is an increasing interest in analyzing structural vulnerability of power grids by means of complex network methodology.

The vulnerability analysis of complex networks is mainly originated from ecological networks studies. It was found that food webs are very robust against random removals but extremely fragile when selective attacks are used [3]. Moreover, the robustness of food webs increases with its connectance which appears independent of species richness and omnivory [4]. After that, the kind of analysis arises in power grids [5][6][7]. It is found that North American power grid [5] and European power grid [6][7] have vulnerable response to the successive removal of nodes and similar to the scale-free network, though the two power grids do not have the topological feature of scale-free network [8][9] that frequency of nodes with connections follows a power-law distribution.

However, the straight application of the complex network method neglects all the specific engineering features of power grids; therefore, the analytical results may be far from real-world power systems, and it seems more appropriate to analyze the structural vulnerability of electrical power grids combining some electrical engineering features with complex networks theory. Following this idea, the metrics of entropic degree [10], electrical betweenness [11] and net-ability [12] were proposed by

introducing some electrical engineering features such as electrical distance, line flow limits and power transmission distribution into the complex networks metrics: *degree* [13], *betweenness* [14] and *efficiency* [15]. Moreover, two new metrics were defined to assess the vulnerability of power grids: *entropic path redundancy* [16] and *survivability* [17]. In this paper, we will use some of these extended metrics to analyze the structural vulnerability of the interconnected power grid made by the simplified power grid of the continental Europe together with the Intergrated Power System and Unified Power System (IPS/UPS) of CIS and Baltic countries¹. Specifically, we investigate the classification of these power grids in terms of the cumulative distribution of entropic degree and extended betweenness; meanwhile, the critical buses and lines are located in each power grid using entropic degree and electrical betweenness. Furthermore, we analyze the resilience to intentional attacks on critical components in each power grid. By means of the structural vulnerability analysis, we expect to know whether the interconnection will be beneficial to these power grids in continental Europe and CIS and Baltic countries.

The rest of this paper is organized as follows: Section 2 provides a review of the extended topological method. In section 3, the interconnected power grid firstly is described and then its vulnerabilities are analyzed. The conclusions are summarized in section 4.

2. Extended Topological Approach for Power Grids

The complex networks theory has been applied in the analysis of electrical power grids. However, the pure topological concepts and metrics disregard the fundamental engineering features of electrical power grids; therefore the analysis resulting from the straight application of the complex network theory cannot capture the reality in power systems. Hence, we extend complex networks metrics by introducing some electrical features in order to practically analyze structural vulnerability of electrical power grid.

Actually, the electrical power grid is a flow-based network where the power flow is transmitted from power plants to consumers; meanwhile, each line has parameters such as flow limits that can be considered as weights to describe the physical constraints on each line. Secondly, buses have different functions in power grids and can be classified as generator buses ($\mathcal{G} \dim\{\mathcal{G}\} = N_{\mathcal{G}}$), transmission buses ($\mathcal{T} \dim\{\mathcal{T}\} = N_{\mathcal{T}}$) and load buses ($\mathcal{D} \dim\{\mathcal{D}\} = N_{\mathcal{D}}$). \mathcal{G} is a set of buses injecting power in the grid while \mathcal{D} is a set of buses withdrawing power from the grid; the set of buses \mathcal{T} includes connection buses that transmit power. Thirdly, it is presumed that each line in power grids has a reference direction. Assume f_{ij} is the

¹ The power grid of CIS countries is called as the Integrated Power System (IPS) while the power grid of Baltic countries is named as the Unified Power systems (UPS) [18].

flow on line l_{ij} , $f_{lij}>0$ means f_{lij} is consistent with the reference direction of line l_{ij} ; otherwise, $f_{lij}<0$. Therefore, it is more feasible to consider the electrical power grid as a weighted and directed model $\mathcal{Y} = \{\mathcal{B}, \mathcal{L}, \mathcal{W}\}$ where \mathcal{B} ($\dim\{\mathcal{B}\} = N_{\mathcal{B}}$) is the set of vertices (or nodes); each vertex can be identified by index i ; \mathcal{L} ($\dim\{\mathcal{L}\} = N_{\mathcal{L}}$) is the set of edges (or links); the edge is identified by l_{ij} that represents a connection between vertex i and vertex j ; \mathcal{W} is set of weights and the weight element w_{ij} in the set \mathcal{W} is associated each line l_{ij} .

Instead of *geodesic distance* d_{ij} which is the number of edges in the shortest path between vertices i and j [19], the distance in a power grid should be considered as the electrical distance between a generation bus i and a load bus j in terms of equivalent impedance Z_g^d [12] which considers the impedance of transmission lines between them. Suppose U_g^d is the voltage between generation bus g and load bus d ; I_g is the current injected at bus g and withdrawn at bus d ($I_g = -I_d$). The equivalent impedance can be expressed as:

$$Z_g^d = \frac{U_g^d}{I_g} \quad (1)$$

Assume that a unit current is injected at bus g and withdrawn at bus d (i.e., $I_g=1$ and $I_d=-1$) while no current is injected or withdrawn at other buses, then equivalent impedance can be calculated as:

$$Z_g^d = \frac{U_g^d}{I_g} = U_g^d = U_g - U_d = (z_{gg} - z_{gd}) - (z_{gd} - z_{dd}) = z_{gg} - 2z_{gd} + z_{dd} \quad (2)$$

where z_{gd} is the g -th, d -th element of the impedance admittance matrix [20].

In complex networks theory, the *degree* k_i , is a basic metric that can measure the criticality of vertex i , and a vertex with higher degree is more important than others. In unweighted networks, the degree of vertex i is the number of the edges connected to the vertex [13]; in a weighted network, the degree of vertex i is defined as the *strength* s_i of the vertex which is the sum of the weights on the edges connecting node i . However, the strength of a vertex cannot take account of the distribution of weights among edges, so the strength metric is unable to effectively identify importance of a bus in a power grid. Therefore, we redefine the *entropic degree* k_i^w of a vertex i by introducing the concept of entropy into the strength metric [10]:

$$k_i^w = (1 - \sum_{j=1}^{N_{\mathcal{B}}} p_{ij} \cdot \log p_{ij}) \sum_{j=1}^{N_{\mathcal{B}}} w_{ij} \quad (3)$$

$$p_{ij} = \frac{w_{ij}}{\sum_{j=1}^{N_B} w_{ij}} \quad (4)$$

where p_{ij} is the normalized weight of the edge l_{ij} connecting vertices i and j , and $\sum_{j=1}^{N_B} p_{ij} = 1$.

The entropic degree has the same role as the degree metric to measure the importance of nodes. Also, the network classification (i.e., homogeneous or heterogeneous networks) can be identified by entropic degree cumulative distribution $P(k^w \geq K^w)$. The distribution is the probability that the entropic degree of a node randomly selected is not smaller than K^w . In a homogeneous network, nodes basically have the similar entropic degree and its cumulative distribution is therefore a Poisson distribution, while in a heterogeneous network, most of nodes have a lower entropic degree, but small nodes, so-called hubs, have higher entropic degree than others and the entropic degree cumulative distribution is more possibly exponential or power law; hence, heterogeneous networks are more sensitive to intentional attacks on hubs but are resilient to randomly removal of nodes.

Betweenness is another measure of the criticality of a vertex or an edge. Betweenness is the number of geodesic paths, connecting whichever pair of vertices, passing through a given vertex or edge in a network [14]. The higher betweenness is, a greater number of geodesic paths passing through the component (vertex or edge) are, and higher betweenness implies a higher criticality of the vertex or the edge. Therefore, the critical components of a network can be identified by ranking the betweenness value of the components in a network.

In the definition of betweenness, it is assumed that a unit of physical quantity is transmitted along the geodesic path between a pair of vertices. However, in a power grid, more than one unit of power is transmitted along the electrical path from a generator to a load. As a consequence, we extend the betweenness metric by considering power transmission capacity C_g^d and Power Transfer Distribution Factors (PTDF).

Because power flowing on a line may be positive or negative, we define positive betweenness and negative betweenness of a line l_{ij} . *Positive betweenness of a line*, $B_e^p(l_{ij})$, is the sum of power flowing through the line along its reference direction when power is transmitted from all pairs of generator and load; on the other hand, *negative betweenness of a line*, $B_e^n(l_{ij})$, is the sum of power flowing through the line against its reference direction when power is transmitted from all pairs of generator and load. As it is impossible that both positive and negative power simultaneously exists on the same line, the *extended betweenness of a line* l_{ij} is the maximum between positive betweenness and negative betweenness of a

line [11].

$$B_e(l_{ij}) = \text{Max}(B_e^p(l_{ij}), |B_e^n(l_{ij})|), l_{ij} \in \mathcal{L} \quad (5)$$

$$B_e^p(l_{ij}) = \sum_{g \in \mathcal{G}} \sum_{d \in \mathcal{D}} C_g^d f_{l_{ij}}^{gd}, \text{ if } f_{l_{ij}}^{gd} > 0$$

$$B_e^n(l_{ij}) = \sum_{g \in \mathcal{G}} \sum_{d \in \mathcal{D}} C_g^d f_{l_{ij}}^{gd}, \text{ if } f_{l_{ij}}^{gd} < 0$$

where $B_e(l_{ij})$ is extended betweenness of a line l_{ij} ; C_g^d is power transmission capacity which is defined as:

$$C_g^d = \min_{l_{ij} \in \mathcal{L}} \left(\frac{P_{l_{ij}}^{\max}}{|f_{l_{ij}}^{gd}|} \right) \quad (6)$$

where $P_{l_{ij}}^{\max}$ is the line flow limit of line l_{ij} ; $f_{l_{ij}}^{gd}$ is the power on line l_{ij} ($l_{ij} \in \mathcal{L}$) for a unit of power injected at generation bus g ($g \in \mathcal{G}$) and withdrawal at load bus d ($d \in \mathcal{D}$), and $f_{l_{ij}}^{gd}$ can be computed as follows:

$$f_{l_{ij}}^{gd} = f_{l_{ij}}^g - f_{l_{ij}}^d, l_{ij} \in \mathcal{L} \quad (7)$$

where $f_{l_{ij}}^g$ and $f_{l_{ij}}^d$ are respectively the l_{ij} -th row, g -th column and the l_{ij} -th row, d -th column of matrix F . Matrix F represent the $N_{\mathcal{L}} \times N_{\mathcal{B}}$ PTDF matrix in which an element $f_{l_{ij}}^v$ represents the change of power on line l_{ij} for a unit of power injected at bus v and withdrawn at the reference bus. If $f_{l_{ij}}^v$ is consistent with the reference direction of line l_{ij} , then $f_{l_{ij}}^v > 0$; otherwise, $f_{l_{ij}}^v < 0$.

The input power of a bus v should be equal to output power of the bus, so the *extended betweenness of a bus* v is the half of sum of power flowing through the lines connecting the bus [11].

$$B_e(v) = \frac{1}{2} \left(\sum_{g \in \mathcal{G}} \sum_{d \in \mathcal{D}} C_g^d \sum_{l_{ij} \in \mathcal{L}^v} |f_{l_{ij}}^{gd}| \right) \quad (8)$$

where \mathcal{L}^v is the set of lines connecting a bus v .

In analyzing the power grid, we use the extended betweenness instead of the classic betweenness to capture the electrical features of the power grid; extended betweenness is exploited as a measure of the importance of components and allows for the classification of the grid in terms of betweenness cumulative distribution ($P(B_e(v) \geq O^v)$ or $P(B_e(l) \geq O^l)$) that expresses the probability that the extended betweenness of a bus or a line, randomly selected, is greater or equal to O^v or O^l .

In order to analyze the performance of a network, *efficiency* is introduced into complex networks theory [15]. In the definition of efficiency, it is assumed that a unit of physical quantity is transmitted along a geodesic path between a pair of vertices. Therefore, similarly to what was done for extended betweenness, we extended the metric of efficiency as *net-ability* by replacing a unit of physical quantity transmitted and geodesic distance with maximum transmission capacity and electrical distance,

respectively [12]:

$$A(\mathcal{Y}) = \frac{1}{N_{\mathcal{G}} N_{\mathcal{D}}} \sum_{g \in \mathcal{G}} \sum_{d \in \mathcal{D}} \frac{C_g^d}{Z_g^d} \quad (9)$$

where $N_{\mathcal{G}}$ and $N_{\mathcal{D}}$ are the number of generation buses and load buses in a power grid, respectively ; Z_g^d is the equivalent impedance between generation bus g and load bus d .

After identifying the importance of network components (buses and lines) based on entropic degree or extended betweenness, the structural vulnerability can be analyzed through removing successively buses or lines according to their decreasing values of entropic degree or extended betweenness while the change of network performance can be quantified as:

$$A^{norm}(\mathcal{Y} - \mathbf{1}) = \frac{A(\mathcal{Y} - \mathbf{1})}{A(\mathcal{Y})} \quad (10)$$

where $A(\mathcal{Y} - \mathbf{1})$ represents the net-ability of power grid after removal of a component.

We would like to point out that we could also remove the buses or lines one by one and compare the drop in net ability; By removing the components successively we can just more easily see which are the most critical ones. This does not mean that during an attack all these elements will be attacked together or that the network can still operate in these conditions. We would also like to stress that the vulnerability analysis of power grids involves structure and operative conditions two sides [16]. The operative conditions refer to various load demands and the corresponding generations of power plants which are distributed in a power grid in terms of power flow while the structure of a power grid is the transmission network that is composed of buses and lines to transfer power from power plants to final users. Comparatively, the operative conditions of a power grid are usually changing due to continuously varying load demands whilst the structure of a power grid is relatively fixed because there are few changes over a long time in a typical configuration such as position of buses, length, impedance and line flow limit of transmission lines after the power grid operates. The outage of a power grid is considered as a result that the vulnerability of both operative conditions and structure simultaneously occurs. In other words, structural vulnerability is the inherent structural weakness of a power grid which is unrelated to operative states, but the weakness could not cause catastrophic consequences till the vulnerable operative conditions reach.

The traditional vulnerability analysis based on AC or DC power flow computation depends on operative states; therefore various operative states will lead to various analysis results. On the contrary, the structural vulnerability analysis by means of complex network method can find out the structural

weakness which inherently exists in a power grid. This paper focuses on the structural vulnerability analysis based on our proposed extended topological method. We will investigate and compare the structural vulnerability of various power grids by successively removing a group of critical buses, which is a typical scenario in structural analysis, though removing buses may be not feasibly considered as contingencies in traditional vulnerability analysis depending on operative conditions such as AC power flow calculation. In this paper, the network performance of a power grid is always evaluated in terms of net-ability, and the critical buses are identified by entropic degree and electrical betweenness. In next section, we will perform the structural investigation in the interconnected power grid between continental Europe, Baltic states and CIS countries.

3. The Vulnerability Analysis of the Interconnected Transmission Power Grid

3.1 The interconnected power grid constituted by the simplified continental Europe and IPS/UPS

In this paper, we create a simplified interconnected power grid made of a simplified continental Europe power grid [21] and an IPS/UPS power grid as shown in Figure 1 and Figure 2, respectively. The IPS/UPS has 713 buses and 943 lines while there are 1254 buses and 1944 lines in the simplified continental Europe power grid as shown in Table 1. In addition, the simplified continental Europe power grid has 17 members of continental Europe power grid as shown in Table 2. The continental European power grid which was the former Union for the Coordination of the Transmission of Electricity (UCTE) power grid is now one of 5 regional group power grids in ENTSO-E [22]. The simplified continental Europe and IPS/UPS power grids are interconnected by 7 interconnection tie lines connecting 7 pairs of buses as shown in Table 3. Figure 3 illustrates the interconnection of the simplified continental Europe and IPS/UPS power grids, and the number of components in the simplified interconnection power grid is shown in Table 1 as well. Table 4, Table 5 and Table 6 give the detailed comparison of bus voltage level, transmission line voltage level and transformer voltage level in the simplified continental Europe power grid and IPS/UPS power grid as well as in the interconnected power grid.

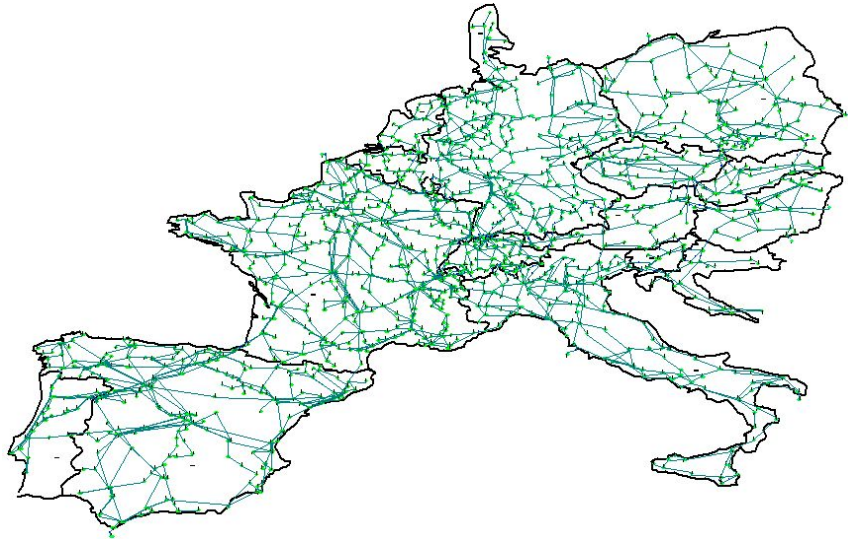


Figure 1 The simplified continental Europe power grid

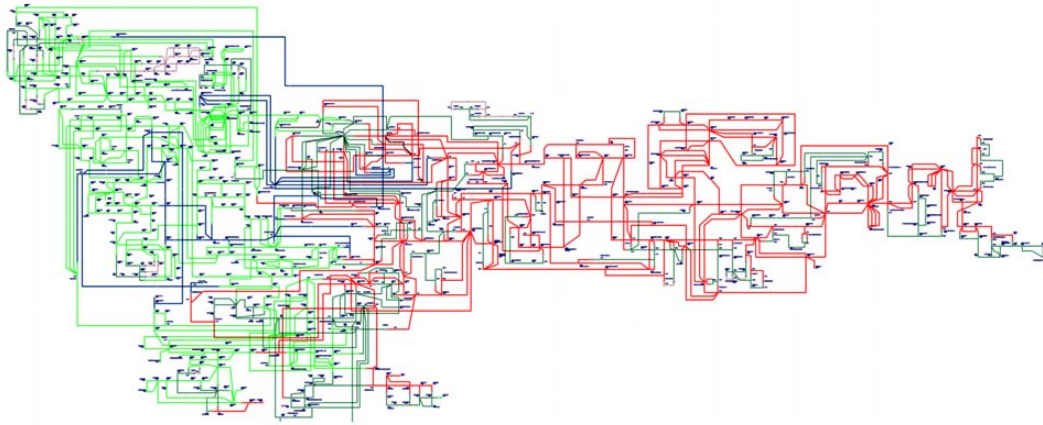


Figure 2 The IPS/UPS power grid

Table 1

Table 2

Table 3

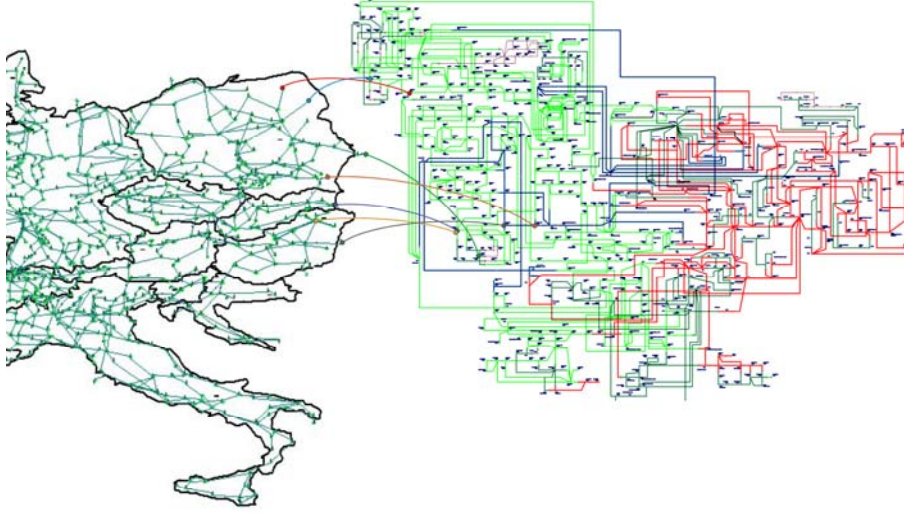


Figure 3 The interconnected power grid of the simplified continental Europe with IPS/UPS

Table 4

Table 5

Table 6

3.2 Analyzing the vulnerability of the interconnected power grid

In this paper, we assess the vulnerability of the interconnected continental Europe - IPS/UPS power grid using entropic degree, extended betweenness and net-ability. These three metrics seem to be able to provide better information than their purely topological counterparts [10][11][12]. Firstly, it is important to spot out the critical components which have higher connectivity or power transmission. After that, these three power grids will be analyzed and compared according to the resilience to intentional attacks on critical components.

The entropic degree is a good indicator of the topological importance of buses in power grids; thus we compute the cumulative distributions of entropic degree in the IPS/UPS power grid, simplified continental Europe power grid and interconnected power grid. We find that these distributions follow three exponential distributions as shown in Figure 4 and the corresponding fitting functions of these distributions are described in Table 7. Similarly, cumulative extended betweenness distributions of buses are also computed in these power grids. These distributions are exponential as shown in Figure 5 and at the same time Table 8 reports the fitting function of distributions of extended betweenness. As a result, these power grids seem to be heterogeneous networks where some buses have higher entropic degree or extended betweenness than others. In other words, there exist some critical buses with higher entropic

degree or extended betweenness in each of the three power grids. In the following structural vulnerability analysis, we can see that these critical buses play an important role in maintaining the global network performance of each power grid in terms of net-ability.

Moreover, it is found that critical buses with respect to high degree are possibly not those buses with high extended betweenness. We can rank the components in descending order of entropic degree and extended betweenness, respectively. Table 9 and Table 10 report the top 10 most critical buses spotted by entropic degree and extended betweenness in these power grids, respectively. As we can see in the two tables, entropic degree gives a different ranking of criticality of buses from extended betweenness. For instance, the rank of bus 1329 in IPS/UPS is the fifth position according to entropic degree whereas the bus is the first in terms of extended betweenness. On the other hand, the most critical bus according to entropic degree is bus 1369 of IPS/UPS which cannot be found in Table 10. Thus, both entropic degree and extended betweenness can provide information about various patterns of multiple attacks on the systems, and so both of them should be considered in the vulnerability analysis. According to Figure 4 and Figure 5, it is possible that the most vulnerable network to intentional attacks is the IPS/UPS network, as both its entropic degree and extended betweenness distributions have steeper slopes, which means that it has a lower connectivity and smaller number of buses with higher power transmission inside the IPS/UPS power grid.

Table 7

Table 8

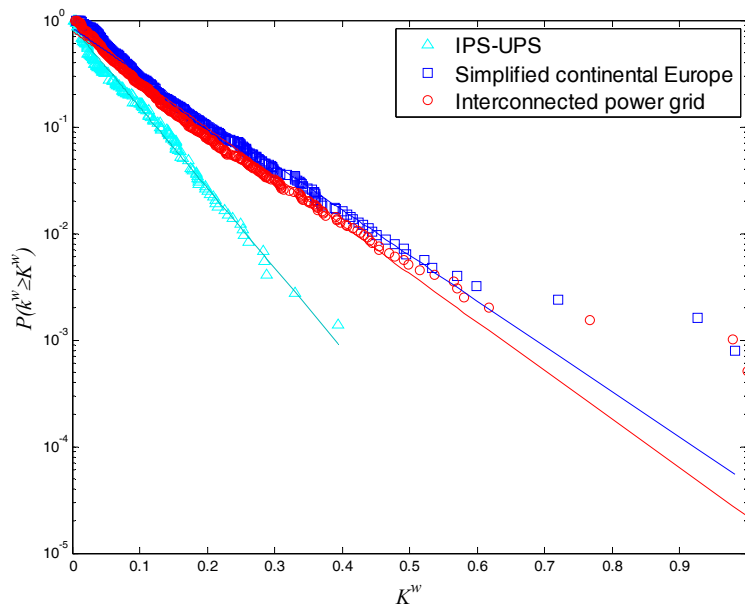


Figure 4 Cumulative distributions of entropic degree in various power grids

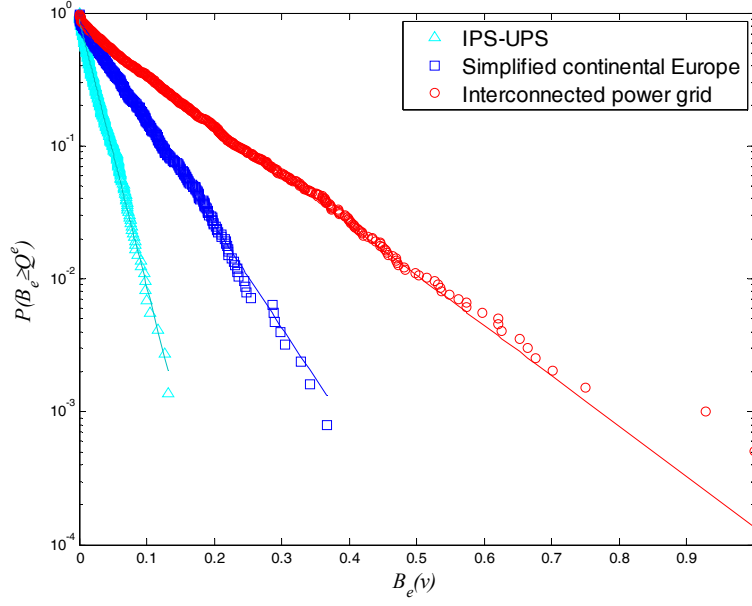


Figure 5 Cumulative distributions of bus extended betweenness in various power grids

Table 9

Table 10

Then, we compare the network performances of three power grids in terms of net-ability $A(\mathcal{Y})$, defined in equation (9), which evaluates the global performance of a power grid according to the ratio between power transmission capability and electrical distance through all pairs of generators and loads in a power grid. The results are reported in Table 11 where it is shown that the net-ability of the interconnected power grid is between the two original separated grids and even lower the average of net-ability of two power grids. From a topological point of view, only the weaker network, the IPS/UPS, appears to benefit from the interconnection, as the simplified continental Europe power grid has still a much better global performance than the interconnected power grid. The reason for this can be found by comparing the averaged power transmission capacity $\overline{C_g^d}$ and averaged electrical distance $\overline{Z_g^d}$ of each power grid as shown in Table 11. $\overline{C_g^d}$ is that sum of the power transmission capacity between each pair of generator and load is averaged by all pairs of generator and load, and $\overline{Z_g^d}$ is that sum of the equivalent impedance between each pair of generator and load is averaged by all pairs of generator and load. Interconnected power grid has smaller $\overline{C_g^d}$ and larger $\overline{Z_g^d}$ than simplified continental Europe, which causes the net-ability of the interconnected power grid is smaller than simplified continental Europe

power grid. As we can see, IPS/UPS power grid with the lowest power transmission capacity bring about that interconnected power grid has a lower transmission capacity; on the other hand, the long distance connection between continental Europe and IPS/UPS increases the electrical distance of interconnected power grid. As a result, increasing the capacity of transmission line in IPS/UPS power grid and reducing the long interconnected distance between continental Europe and IPS/UPS power grids could effectively enhance the whole performance of interconnected power grid.

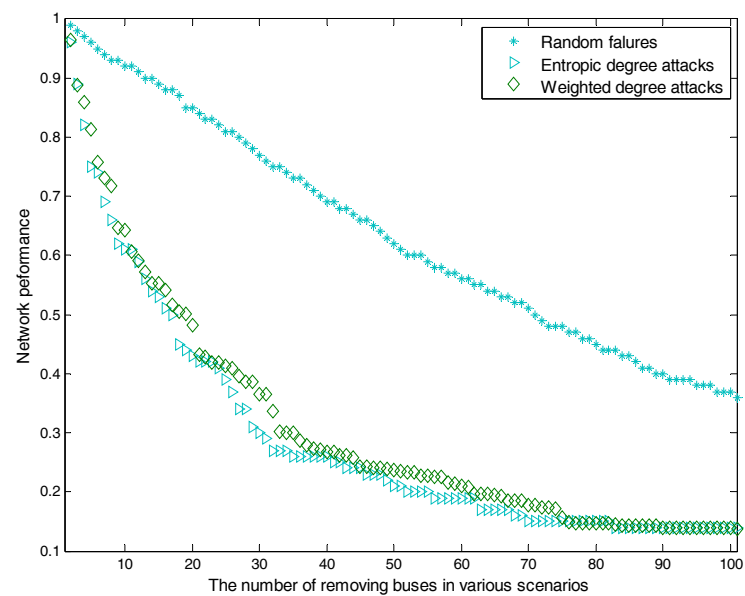
Table 11

Furthermore, we investigate the effect on global performance of randomly and intentionally attacking buses of each power grid, using $A^{\text{norm}}(\mathbf{y}-1)$ defined in equation (10). The reason why we choose buses instead of lines as random and selective failures in each power grid is that we attempt to analyze the vulnerability of these three power grids in the worst case from a structural angle. As we know, the deletion of a bus has a more serious consequence in a power grid than a line from a structural perspective since the attack on a bus will damage all lines connecting the bus rather than only a line.

Moreover, in the case of the intentional attack, a set of critical buses are generally selected instead of all buses [5]. As a result, the intentional attacks in each power grid are the most critical 100 buses in terms of descending order ranked by entropic degree and electrical betweenness, respectively; then, these buses are successively removed from each power grid to analyze the structural vulnerability. Also, we randomly choose 100 buses as random attacks in each power grid in order to compare the results with that of intentional attacks. For these random attacks, 100 buses are randomly selected and then removed successively from each power grid in a simulation and then the net-ability is evaluated by averaging 100 simulations of the random failures in each power grid. Besides, the first 100 most critical buses are chosen as selective failures according to the descending order ranked by strength and topological betweenness metrics, respectively, because we expect to further demonstrate the superiority of extended topological metrics to topological metrics in large-scale power grids.

When these power grids are attacked either randomly or deliberately, we monitor the change of network performance $A^{\text{norm}}(\mathbf{y}-1)$ as a function of the number of removed buses shown in Figure 6-Figure 8. As we see from these figures, initially, the net-ability of each power grid is 100% because of no attack on buses has been performed in these power grids. However, the net-ability decreases dramatically with increasing number of the removed buses since the changed power transmission paths cause the growing electrical distance and the reducing power transmission capacity. Especially, it can be seen that these power grids are heterogeneous networks which are sensitive to intentional attacks but relatively robust to

random failures as in each power grid net-ability drops much more steeply in selective failures cases. Besides, it seems that these 100 critical buses identified in terms of topological or extended topological metrics are indeed significant for each power grid because less 50% of initial net-ability can be maintained after removing these critical buses from each power grid. Particularly, in the IPS/UPS power grid, less 20% of its original net-ability can be retained after these critical buses are deleted. Comparatively, though the selective failures scenario for entropic degree shows no much clear superiority to the strength metric, extended topological metrics are generally better than topological metrics. The reason for this is that net-ability of each power grid drops more quickly when buses are intentionally attacked in terms of extended topological metrics.



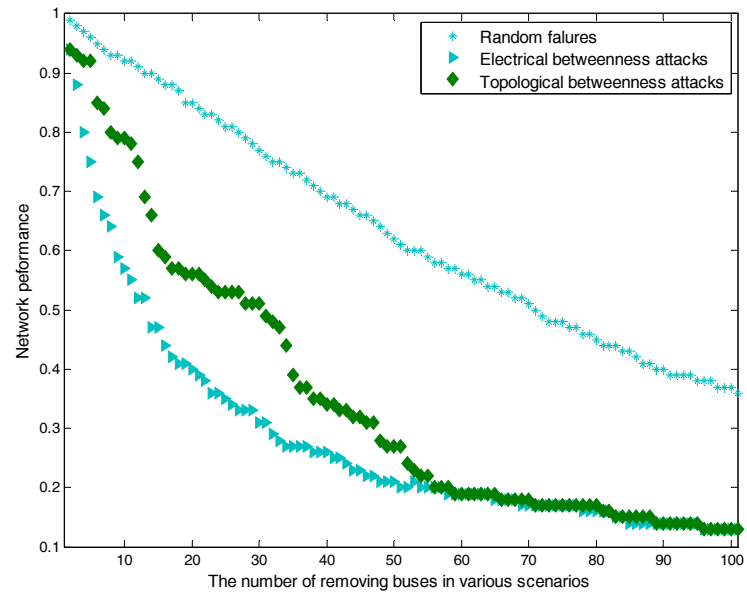
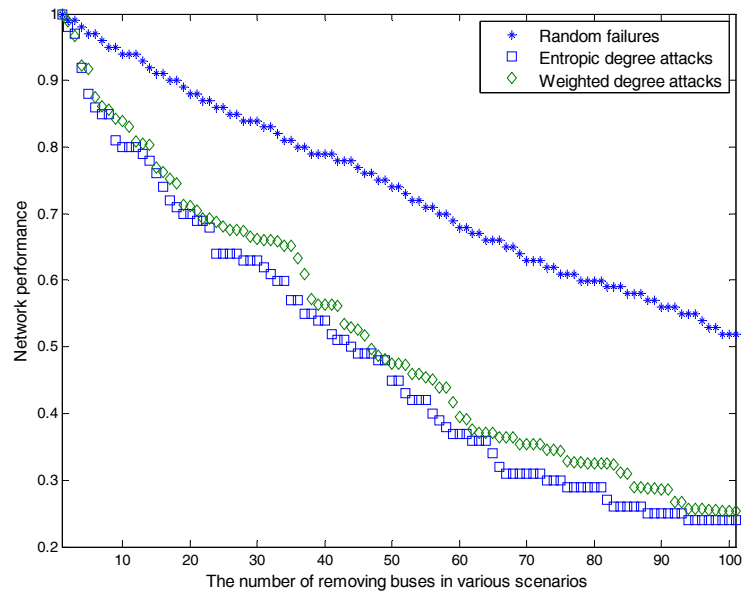


Figure 6 Relative network performance of the IPS/UPS power grid after removing 100 buses in various scenarios



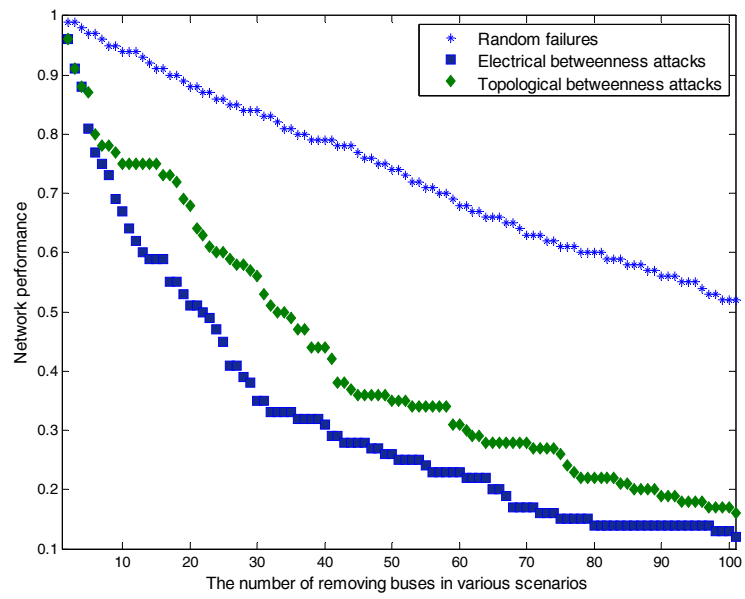
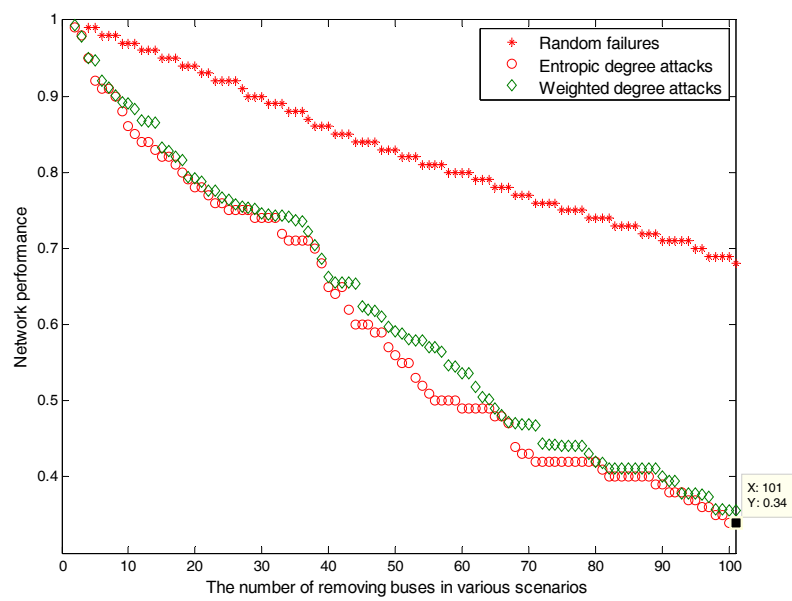


Figure 7 Relative network performance of the simplified continental Europe power grid after removing 100 buses in various scenarios



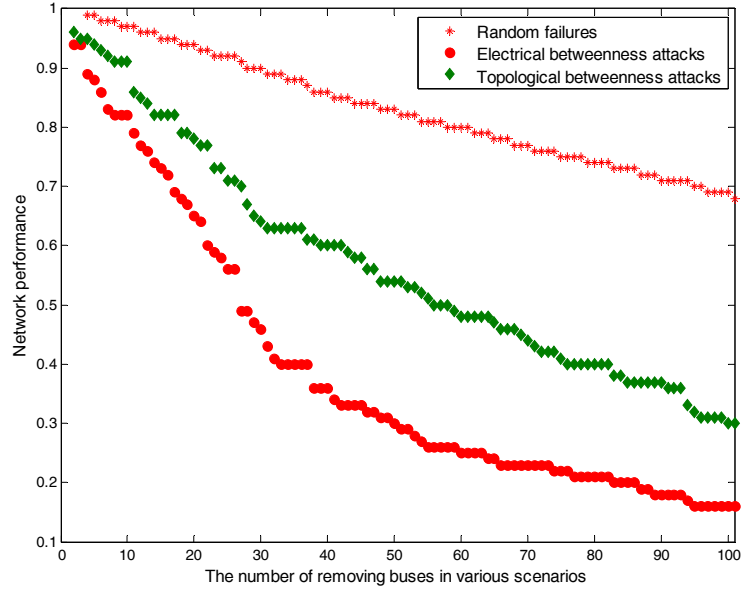


Figure 8 Relative network performance of the interconnected power grid after removing 100 buses in various scenarios

At the same time, we compare the vulnerability of these power grids in the case where only critical buses are deliberately deleted in terms of entropic degree (shown in Figure 9) or extended betweenness (shown in Figure 10). As can be seen from Figure 9, the IPS/UPS power grid is more vulnerable than other power grids because the IPS/UPS power grid loss its net-ability faster than other power grids when the 100 critical buses for entropic degree are removed successively. The reason for this can be found in Figure 4 where the IPS/UPS power grid has smaller probability that buses have higher connectivity in terms of entropic degree than other power grids. In other words, compared with the simplified continental Europe and interconnected power grid, the IPS/UPS power grid has smaller number of buses with higher connectivity. Meanwhile, this power grid has also smaller number of transmission lines to construct power transmission paths connecting generators and loads. As a result, after removal of 100 critical buses from the power grid, few buses with higher connectivity and power transmission paths could remain in the IPS/UPS power grid so that its net-ability drops quickly. On the other hand, although Figure 4 illustrates that the simplified continental Europe and interconnected power grids basically have the same probability of buses with higher connectivity, Figure 9 shows that the interconnected power grid is more robust. The explanation for this is that the interconnected power grid has more generators, loads and transmission lines than the simplified continental Europe power grid; therefore, after the same number of critical buses is removed from each of the two power grids, the interconnected power grid could have more power transmission paths to preserve its net-ability.

Similarly, as for attacks on buses ordered in terms of bus extended betweenness, we can see from Figure 10 that the IPS/UPS power grid is also more vulnerable than other power grids, after the 100 critical buses are removed successively from each power grid. This is due to the fact that the bus extended betweenness represents the power transmitted through a bus. Figure 5 shows that the IPS/UPS power grid has a smaller number of buses transmitting a larger amount of power than other power grids whilst, by contrast, the interconnected power grid has the largest number of this kind of buses among the three power grids. Therefore, after removing the 100 critical buses from each power grid, a few buses which can transmit higher power remain in the IPS/UPS power grid to maintain its net-ability while interconnected power grid still has larger number of remaining buses which can transmit more power so that the interconnected power grid can still stay at the higher level of net-ability.

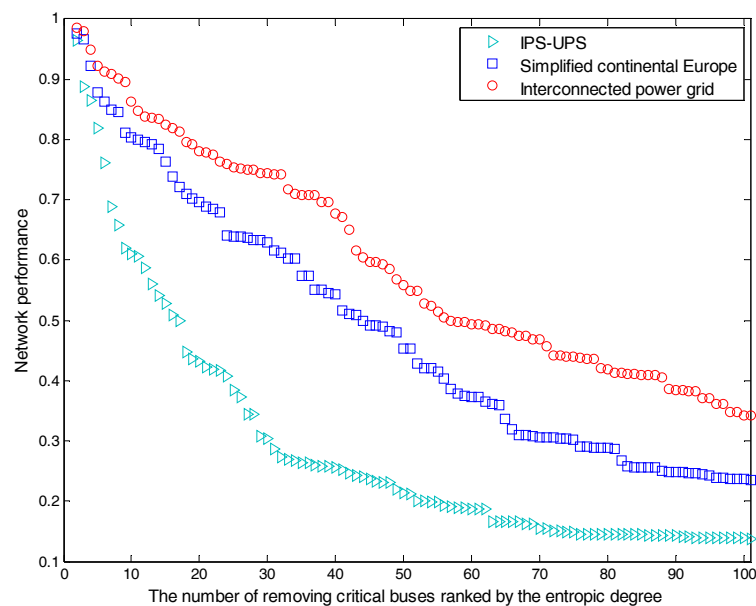


Figure 9 Comparison of relative network performance in various power grids after removing critical buses ranked by entropic degree

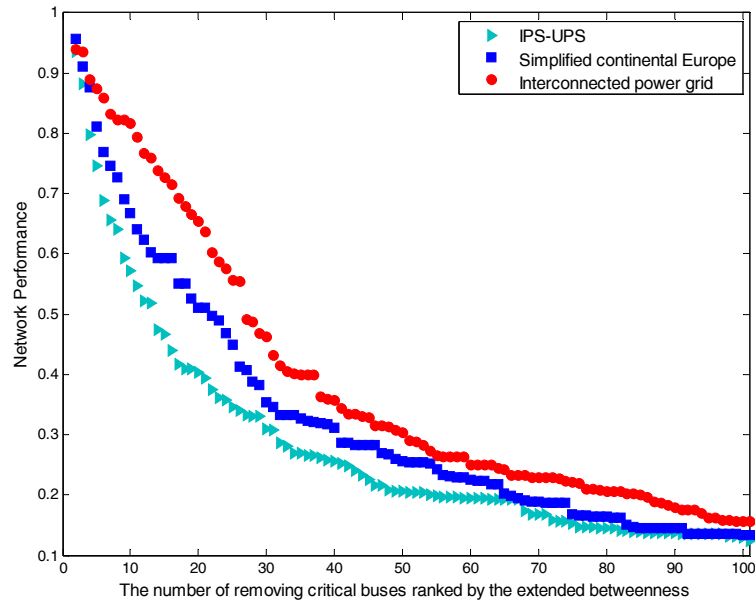


Figure 10 Comparison of relative network performance in various power grids after removing critical buses ranked by extended betweenness

Figure 9 and Figure 10 show the change of net-ability in each power grid when each power grid is intentionally attacked. The changing net-ability of the three power grids are normalized by the original performance of each power grid, respectively. However, the difference of original net-ability for each power grid that was presented in Table 11 is not showed in Figure 9 and Figure 10 because of the chosen normalization mode. For this reason, we also show the changed net-ability in Figure 11 and Figure 12 where the net-ability of each power grid is normalized by the largest original net-ability among the three power grids (i.e., the simplified continental Europe power grid) when each power grid is intentionally attacked in terms of either entropic degree or extended betweenness. It can be seen from Figure 11 and Figure 12 that the interconnected power grid always maintains a net-ability that mediates the performances of the two separate power grids in the whole attacking process. Moreover, we can observe that the loss of net-ability in the interconnected power grid is slower than in the other two power grids when 100 critical buses are removed successively. That is, the interconnection of the two original power grids with a small number of tie lines creates a network which has an average net-ability between two original power grids but is more robust than two separate power grids under intentional attacks.

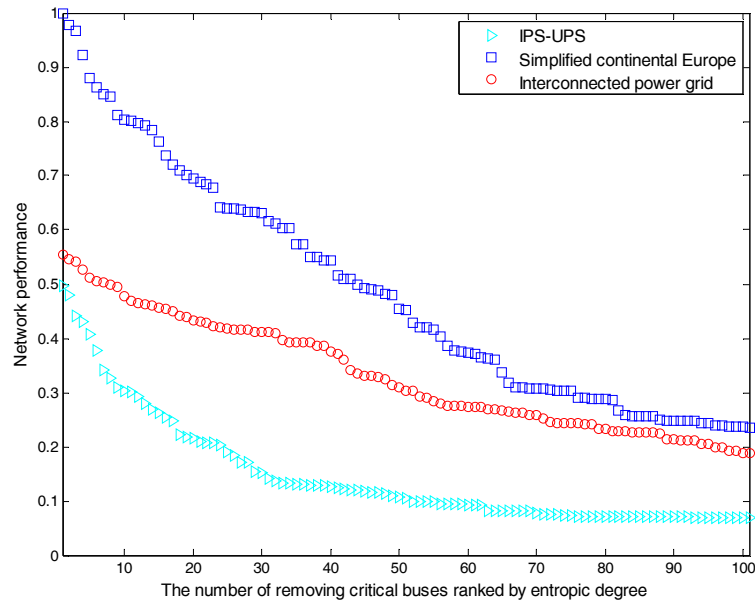


Figure 11 Comparison of network performance in various power grids after removing critical buses ranked by entropic degree

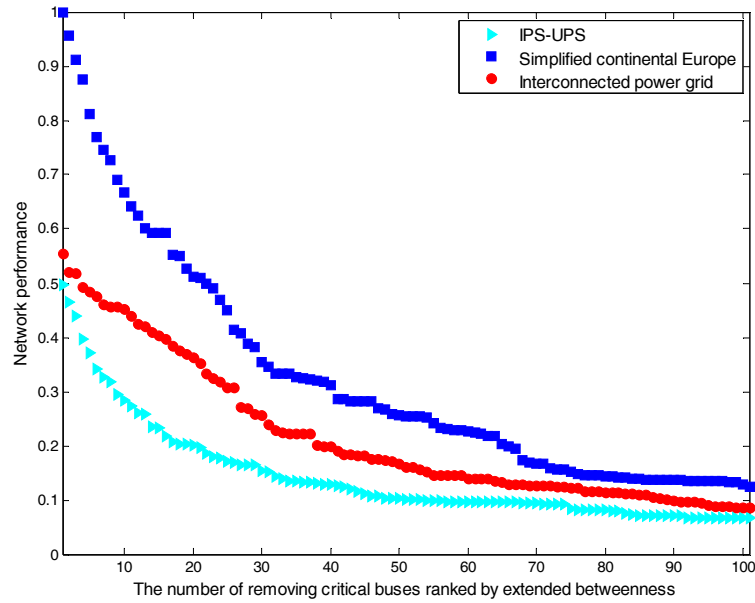


Figure 12 Comparison of network performance in various power grids after removing critical buses ranked by extended betweenness

4. Conclusions

As the interconnection between European and Russian power grids will create the largest interconnected power grid in the world, analyzing the vulnerability of the large-scale power grid is necessary to maintain its reliable and secure operation. The vulnerability of power grids can be analyzed from operative statuses and structure two sides. In other words, the outage of a power grid is considered as a result that the vulnerability of both two sides simultaneously occurs. The structural vulnerability is

the inherent topological weakness of a power grid which is independent of varying operative states.

In this paper, we analyze the structural vulnerability of the large-scale interconnected transmission network connecting continental Europe and CIS and Baltic countries using our proposed extended topological method. Similar to studies in North American and European power grid, our analysis shows that each investigated power grid is vulnerable to selective failures of critical buses but robust to random failures, though each single power grid displays less skewed exponential entropic degree distribution than a power-law distribution. Moreover, the different response of each power grid to selective and random failures is independent of the extended measures (i.e., entropic degree or electrical betweenness). Comparatively, when these three power grids are deliberately attacked, the interconnected power grid is the most robust among the three power grids while the IPS/UPS power grid is the most vulnerable. Meanwhile, when the network performances of these power grids are evaluated in terms of net-ability measure, the interconnection of the simplified continental Europe and IPS/UPS power grids can only improve the network performance of the IPS/UPS rather than both of continental Europe and IPS/UPS power grids. The reason for this is that the lower line transmission capacity in IPS/UPS power grid than continental Europe reduces its averaged power transmission capacity and so the interconnected power grid has the averaged lower transmission capacity as well; on the other hand, the long distance connection between continental Europe and IPS/UPS increases the electrical distance in the interconnected power grid. Consequently, increasing the capacity of transmission line in IPS/UPS power grid and reducing the long interconnected distance between continental Europe and IPS/UPS power grids could not only effectively enhance the whole performance of interconnected power grid, but also increase the structural robustness of the simplified continental Europe and IPS/UPS power grids.

5. Symbols

\mathcal{U}	Undirected and unweighted graph, $\mathcal{U} = \{\mathcal{B}, \mathcal{L}\}$
\mathcal{B}	Set of vertices, $\mathcal{B} = \{\dots, i, \dots\}$, $\dim\{\mathcal{B}\} = N_{\mathcal{B}}$, $\mathcal{B} = \mathcal{G} \cup \mathcal{D} \cup \mathcal{T}$
\mathcal{G}	Set of generation buses, $\mathcal{G} \subseteq \mathcal{B} = \{\dots, g, \dots\}$, $\dim\{\mathcal{G}\} = N_{\mathcal{G}}$
\mathcal{D}	Set of load buses, $\mathcal{D} \subseteq \mathcal{B} = \{\dots, d, \dots\}$, $\dim\{\mathcal{D}\} = N_{\mathcal{D}}$
\mathcal{T}	Set of transmission buses, $\mathcal{T} \subseteq \mathcal{B} = \{\dots, t, \dots\}$, $\dim\{\mathcal{T}\} = N_{\mathcal{T}}$
\mathcal{L}	Set of edges, $\mathcal{L} = \{\dots, l_{ij}, \dots\}$, $\dim\{\mathcal{L}\} = N_{\mathcal{L}}$, $i \neq j \in \mathcal{B}$
\mathcal{L}^V	Set of edges connecting vertex v , $\mathcal{L} = \{\dots, l_{iv}, \dots, l_{vj}, \dots\}$
\mathcal{W}	Set of weights, $\mathcal{W} = \{\dots, w_{ij}, \dots\}$, $i \neq j \in \mathcal{B}$
k_i	Degree of a vertex i
s_i	Strength of a vertex i
d_{ij}	Geodesic distance between vertex i and vertex j

p_{ij}	Normalized weight of the edge l_{ij}
k_i^w	Entropic degree of a vertex i
$P(k^w \geq K^w)$	Entropic degree cumulative distribution
U_g^d	Voltage between generation bus g and withdraw at load bus d
I_g	Current injected at bus g and withdrawn at bus d
Z_g^d	Equivalent impedance for injection at generation bus g and withdraw at load bus d .
z_{gd}	The g -th, d -th element of the bus impedance matrix
C_g^d	Power transmission capacity from generator bus g to load bus d
f_{lij}	Flow on line l_{ij}
\mathbf{F}	PTDF matrix, $\dim(\mathbf{F}) = N_{\mathcal{E}} \times N_{\mathcal{B}}$
f_{lij}^g	The l_{ij} -th, row g -th column of matrix \mathbf{F}
f_{lij}^{gd}	Flow on line l_{ij} for a unit of power injected at generation bus g and withdrawal at load bus d
P_{lij}^{max}	Line flow limit of line l_{ij}
$B_e^p(l_{ij})$	Positive betweenness of a line l_{ij}
$B_e^n(l_{ij})$	Negative betweenness of a line l_{ij}
$B_e(l_{ij})$	Extended betweenness of a line l_{ij}
$B_e(v)$	Extended betweenness of a bus v
$P(B_e(v) \geq O^v)$	Cumulative distribution of bus extended betweenness
$P(B_e(l) \geq O^l)$	Cumulative distribution of line extended betweenness
$A(\mathcal{Y})$	Net-ability of network \mathcal{Y}
$A(\mathcal{Y}-1)$	Net-ability of power grid after removal of a component
$A^{norm}(\mathcal{Y}-1)$	Normalized net-ability of a network \mathcal{Y} after removal of a component

6. References

- [1] Hines P., Blumsack S. A centrality measure for electrical networks. Hawaii International Conference on System Sciences Proceedings of the 41st Annual, Waikoloa HI, 2008; 185. DOI: 10.1109/HICSS.2008.5
- [2] Baldick, R., Chowdhury, B., Dobson, I., Zhaoyang Dong, et al. Initial review of methods for cascading failure analysis in electric power transmission systems IEEE PES CAMS task force on understanding, prediction, mitigation and restoration of cascading failures. Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century, 2008 IEEE; 1-8. DOI: 10.1109/PES.2008.4596430
- [3] Solé, R. V., J. M. Montoya. Complexity and fragility in ecological networks. Proc.R. Soc. Lond. B 2001; 268: 2039-2045. DOI: 10.1098/rspb.2001.1767
- [4] Jennifer A. Dunne, Richard J. Williams, Neo D. Martinez. Network structure and biodiversity loss in food webs: robustness increase with connectance. Ecology Letters 2002. 5: 558-567. DOI: 10.1046/j.1461-0248.2002.00354.x
- [5] R. Albert, I. Albert, G. L. Nakarado. Structure vulnerability of the North American power grid. Physical Review E 2004; 69: 025103. DOI: 10.1103/PhysRevE.69.025103
- [6] Marti Rosas-Casals, Sergi Valverde, Ricard V. Sole. Topological vulnerability of the European power grid under Errors and Attacks. International Journal of Bifurcation And Chaos 2007; 17: 2465-2475. DOI: 10.1142/S0218127407018531

- [7] Ricard V. Solé, Martí Rosas-Casals, Bernat Corominas-Murtra, Sergi Valverde. Robustness of the European power grids under intentional attack. *Physical Review E* 2008; 77: 026102. DOI: 10.1103/PhysRevE.77.026102
- [8] A. L. Barabási, R. Albert. Emergence of scaling in random networks. *Science* 1999; 286: 509-512. DOI: 10.1126/science.286.5439.509
- [9] A. L. Barabási, R. Albert, H. Jeong. Mean-field theory for scale-free random networks. *Physica A* 1999; 272: 173-187. DOI: 10.1016/S0378-4371(99)00291-5
- [10] E. Bompard, R. Napoli, F. Xue. Analysis of structural vulnerability in power transmission grid. *International journal of critical infrastructure protection* 2009; 2: 5-12. DOI:10.1016/j.ijcip.2009.02.002
- [11] Ettore Bompard, Di Wu, Fei Xue. The concept of betweenness in the analysis of power grid vulnerability. *Complexity in Engineering* 2010; 52-54. DOI: 10.1109/COMPENG.2010.10
- [12] S. Arianos, E. Bompard, A. Carbone, F. Xue. Power grid vulnerability: a complex network approach. *Chaos* 2009; 19: 013119. DOI: 10.1063/1.3077229
- [13] M. E. J. Newman. The structure and function of complex network. *SIAM Review* 2003; 45: 167-256. DOI:10.1137/S003614450342480
- [14] L. C. Freeman. A set of measures of centrality based on betweenness. *Sociometry* 1977; 40: 35-41.
- [15] V. Latora, M. Marchiori. Efficient behavior of small-world networks. *Physical Review Letters* 2001; 87 : 198701. DOI: 10.1103/PhysRevLett.87.198701
- [16] E. Bompard, R. Napoli, F. Xue. Extended topological approach for the assessment of structural vulnerability in transmission networks. *IET Generation Transmission & Distribution* 2010; 4: 716-724. DOI: 10.1049/iet-gtd.2009.0452
- [17] F. Xue. Comprehensive models for security analysis of critical infrastructures as complex systems. Delf: Next Generation Infrastructures Foundation, 2009; ISBN: 978-90-79787-05-0.
- [18] B. Ayuev, P. Erokhine, Y. Kulikov. IPS/UPS wide area measuring system. *CIGRE*, 41st Session, 2006.
- [19] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, D.-U. Hwang. Complex networks: Structure and dynamics. *Physics Reports* 2006; 424: 175-308. DOI: 10.1016/j.physrep.2005.10.009
- [20] J. Duncan Glover, Mulukutla Sarma; *Power System Analysis and Design*, second edition, PWS Publishing Company: Boston, 2008.
- [21] Qiong Zhou, Janusz W. Bialek. Approximate Model of European Interconnected System as a Benchmark System to Study Effects of Cross-Border Trades. *IEEE Transactions on Power Systems*, 2005; 20: 782-788. DOI: 10.1109/TPWRS.2005.846178
- [22] <https://www.entsoe.eu/system-operations/regional-groups/>

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Table 1 Number of components in the simplified continental Europe, IPS/UPS and interconnected power grids

	Simplified continental Europe	IPS/UPS	Interconnected power grid
Bus	1254	713	1963
Line	1944	943	2890
Transformer	0	210	210
Generator	378	399	777
Load	896	547	1443

Table 2 Members of the simplified continental Europe power grid

Member countries of continental Europe power grid [22]	Member countries of the simplified continental Europe power grid
Austria	√
Belgium	√
Bosnia and Herzegovina	
Bulgaria	
Croatia	√
Czech Republic	√
Denmark (west)	√
France	√
FYROM	
Germany	√
Greece	
Hungary	√
Italy	√
Luxemburg	√
Montenegro	
Netherlands	√
Poland	√
Portugal	√
Romania	
Serbia	
Slovakia	√
Slovenia	√
Spain	√
Switzerland	√

Table 3 Interface tie lines of the simplified continental Europe with IPS/UPS

Bus name	Country	Bus name	Country
PL-79	Poland	HAES-750	Ukraine
SK-6	Slovakia	BuTES-3	Ukraine
H-5	Hungary	BuTES-3	Ukraine
H-12	Hungary	ZUkr750	Ukraine
PL-2	Poland	ALITUS	Lithuania
PL-8	Poland	ROSS'	Belarus
PL-51	Poland	DTES-2	Ukraine

Table 4 Bus voltage level of the simplified continental Europe, IPS/UPS and interconnected power grids

Voltage [kV]	750	500	400	380	330	220	150	120	110	70	27	Total
Simplified continental Europe	0	0	0	1254	0	0	0	0	0	0	0	1254
IPS/UPS	25	199	3	0	252	205	0	0	29	0	0	713
Interconnected power grid	25	199	3	1250	252	205	0	0	29	0	0	1963

Table 5 Line voltage level of the simplified continental Europe, IPS/UPS and interconnected power grids

Voltage [kV]	750	500	400	380	330	220	150	120	110	70	27	Total
Simplified continental Europe	0	0	0	1944	0	0	0	0	0	0	0	1944
IPS/UPS	26	311	1	0	366	218	0	0	21	0	0	943
Interconnected power grid	26	311	1	1947	366	218	0	0	21	0	0	2890

Table 6 Transformer voltage level of the simplified continental Europe, IPS/UPS and interconnected power grids

Ratio	750/500	750/330	500/400	500/330	500/220	500/110	400/330	330/330	330/220	330/110	220/110	Total
Simplified continental Europe	0	0	0	0	0	0	0	0	0	0	0	0
IPS/UPS	10	17	1	7	104	1	1	1	35	15	18	210
Interconnected power grid	10	17	1	7	104	1	1	1	35	15	18	210

Table 7 Cumulative distribution functions of entropic degree in various power grids

Power grid	$P(k'' \geq K'')$
IPS/UPS	$0.8678 \exp(-17.394k'')$
Simplified continental Europe	$0.8474 \exp(-9.829k'')$
Interconnected power grid	$0.7953 \exp(-10.482k'')$

Table 8 Cumulative distribution functions of bus extended betweenness in various power grids

Power grid	$P(B_e(v) \geq O'')$
IPS/UPS	$0.8981 \exp(-46.25B_e(v))$
Simplified continental Europe	$0.8872 \exp(-17.764B_e(v))$
Interconnected power grid	$0.8368 \exp(-8.7251B_e(v))$

Table 9 The 10 most critical bus IDs for entropic degree in various power grids

Rank	IPS/UPS	Simplified continental Europe	Interconnected power grid
1	1369	396	396
2	1309	105	105
3	1407	427	427
4	1333	407	407
5	1329	364	364
6	1565	466	199
7	1428	199	466
8	1300	1054	1174
9	1531	1102	1054
10	1424	151	372

Table 10 The 10 most critical bus IDs for extended betweenness in various power grids

Rank	IPS/UPS	Simplified continental Europe	Interconnected power grid
1	1329	427	1858
2	1333	407	1181
3	1565	302	1813
4	1318	523	1325
5	1309	666	1832
6	1885	559	1054
7	1314	486	1840

8	1566	458	1750
9	1287	932	1185
10	1365	886	427

Table 11 Comparison of network performances of various power grids

	IPS/UPS	Simplified continental Europe	Interconnected power grid
\overline{C}_g^d	787.39	1755.87	1177.19
\overline{Z}_g^d	0.1205	0.1219	0.1499
Net-ability	11991.91	24119.11	13348.17